COLLECTIVE BEHAVIOUR OF LINEAR PERTURBATION WAVES OBSERVED THROUGH THE ENERGY DENSITY SPECTRUM

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Summary The concept of collective behaviour for ensembles of physical and biologycal entities is common practice in science, see for instance [1, 2]. In hydrodynamics, it is almost confined into the context of turbulent systems. Here, we propose to export it into the framework of small perturbation dynamics of stable and unstable flows. With the aim to understand whether the nonlinear interaction among different scales in fully developed turbulence can affect the energy spectrum, and to quantify the level of generality on the value of the energy decay exponent of the inertial range, we consider the state that precedes the onset of instability and transition to turbulence. In this condition, the system is constituted by multiple spatial and temporal scales, and is subject to all the processes of the linearized perturbative Navier-Stokes equations: linearized convective transport, linearized vortical stretching, and molecular diffusion. With the important exception of the nonlinear interaction, these features are the same as those characterizing the turbulent state. The linear transient dynamics of three-dimensional perturbations, which is governed by the initial-value problem related to the linearized perturbative Navier-Stokes equations, is very complicated and shows a great variety of different behaviours, not a priori predictable. We ask whether the linearized perturbative system is able to show a power-law scaling for the energy spectrum in an analogous way to the Kolmogorov argument.

We determine the decay exponent of the energy spectrum for arbitrary longitudinal and transversal perturbations acting on a typical shear flow, i.e. the bluff-body wake. Then, we compare the energy spectrum of the linearized perturbative system with the well-known -5/3 Kolmogorov power-law scaling. We observe, for both longitudinal and transversal perturbative waves, a decay rate of $-5/3$ in the intermediate range ($2 < k < 100$), while the energy decays more rapidly for larger wavenumbers ($k > 100$). So far, it seems that the nonlinear interaction is not the main factor responsible of the specific value of the -5/3 decay exponent in the energy spectrum and the spectral power-law scaling of inertial waves is a general dynamical property of the Navier-Stokes equations, valid also for a general small perturbation which lives in the linearized system.

INTRODUCTION

A fundamental notion in the phenomenology of turbulence (in the sense of Kolmogorov 1941) is that a power-law scaling with an exponent close to -5/3 is observed for the energy spectrum over a quite large range of a few decades of wavenumber. This interval is called the inertial range since, at such wavenumbers, the dynamics of the Navier-Stokes equations is dominated by the inertia terms [3]. It is a common criterium for the successful production of a fully developed homogeneous turbulent field to verify that the energy spectrum has such a scaling in the inertial range [4].

We propose to study how the energy spectrum – evaluated as the wavenumber distribution of the perturbation kinetic energy density in asymptotic condition – behaves [5] and to compare it with the exponent of the fully developed turbulent state. The energy spectrum behaviour of the perturbed system is studied using the initial-value problem formulation. The base flow is approximated at a fixed longitudinal station, $x_0 = 50$, through an analytical expansion solution [6] of the Navier-Stokes equations. The Reynolds number is set to a value of 30, in order to consider stable evolutive configurations. The viscous perturbative equations are written in terms of the vorticity and the transversal velocity and then transformed through a Laplace-Fourier decomposition [7, 8] in the plane (x, z) which is normal to the base flow plane (x, y). We define k as the polar wavenumber, $\alpha_r = kcos(\phi)$ as the wavenumber in x direction, $\gamma = ksin(\phi)$ as the wavenumber in z direction, ϕ as the angle of obliquity with respect to the physical plane, and α_i as the spatial damping rate in x direction. The measure of the perturbation growth can be defined through the disturbance kinetic energy density in the plane (α, γ):

$$e(t; \alpha, \gamma) = \int_{-y_d}^{+y_d} (|\hat{u}|^2 + |\hat{v}|^2 + |\hat{w}|^2) dy = \frac{1}{|\alpha^2 + \gamma^2|} \int_{-y_d}^{+y_d} \left(\left| \frac{\partial \hat{v}}{\partial y} \right|^2 + |\alpha^2 + \gamma^2| |\hat{v}|^2 + |\hat{\omega}_y|^2 \right) dy, \qquad (1)$$

where $\hat{u}$, $\hat{v}$ and $\hat{w}$ are the components of the perturbation velocity, $\hat{\omega}_y$ is the transversal vorticity, while $2y_d$ is the extension of the spatial numerical domain. The amplification factor $G(t)$ can be introduced in terms of the normalized energy density, $G(t; \alpha, \gamma) = e(t; \alpha, \gamma)/e(t = 0; \alpha, \gamma)$.

RESULTS

The appearance of different temporal scales associated to the different perturbation wavelengths (see Fig. 1a) suggests that a self-similarity approach should be adopted to describe the temporal evolution. A continuous instataneous normalization can be used by defining $t^* = t/\tau$, with $\tau = G(t)/|dG/dt|$. In Fig. 1, the amplification factor, G, is reported as a function of both t and t^* for the a group of perturbations with $k \in [0.7, 100]$. It should be noted that a subset of intermediate-short waves ($k \in [6, 100]$) showing self-similarity features can be observed (see Fig. 1b). Assuming that for this range the amplification factor distribution is scale-invariant [3], then $G(\lambda t) = \lambda^h G(t)$, with h unique. It can be observed, that

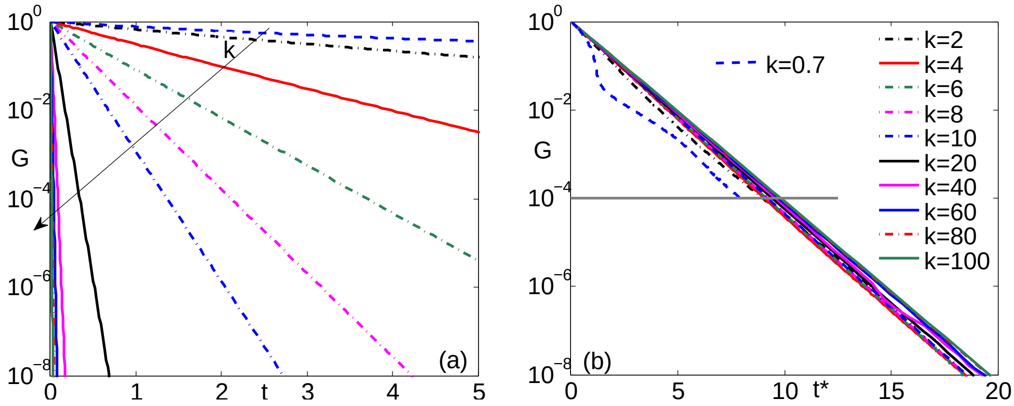

Figure 1. Tentative temporal scaling. (a) G as function of t; (b) G as function of the normalized variable $t^* = t/\tau$, with $\tau = G(t)/|dG/dt|$. Re=30, $x_0 = 50$ $\phi = 0$, symmetric inputs, $k = 0.7, 2, 4, 6, 8, 10, 20, 40, 60, 80, 100$.

$G(t^*) = G\left(\dfrac{t}{G(t)/|dG/dt|}\right) \approx \dfrac{G(t)}{\tau} = |dG/dt|$, so that $\lambda = 1/\tau$ and $h = 1$. The energy spectrum is then evaluated when the temporal variation of $G(t)$ of each wave is crossing a small threshold, e. g. $|dG/dt| < \epsilon = 10^{-4}$ (see the horizontal grey line in Fig. 1b). By recalling that $G(t^*) \approx |dG/dt|$ in the intermediate range, when ϵ is small enough, the perturbation system can be considered in its asymptotic state. Every wavelength shows a characteristic temporal scale, $\tau_\epsilon(k)$, for which $|dG/dt| < \epsilon = 10^{-4}$. This allows to determine a value of $G(t)$ and the corresponding distribution over the wavenumber range. In the interval $k \in [0.5, 500]$ we considered two angles of obliquity $\phi = 0, \pi/2$, symmetric and asymmetric initial conditions in terms of the transversal velocity $\hat{v}$, while the transversal vorticity $\hat{\omega}_y$ is initially zero.
The normalized energy density G (blue and black circles for symmetric and asymmetric initial conditions, respectively) is shown – as function of the polar wavenumber k – in parts (a) and (b) of Fig. 2 for longitudinal ($\phi = 0$) and transversal ($\phi = \pi/2$) perturbative waves, respectively. As a reference, the $-5/3$ slope is shown in red. For both longitudinal and transversal perturbative waves we observe an intermediate range of about two decades of wavenumbers ($k \in [6, 100]$) for which a power-law scaling close to -5/3 exists. For smaller wavelengths ($k > 100$) the decay is slightly faster. For $k < 2$ a slower decay is visible for the asymmetric pertubations. Elsewhere, symmetric and asymmetric perturbations almost coincide.

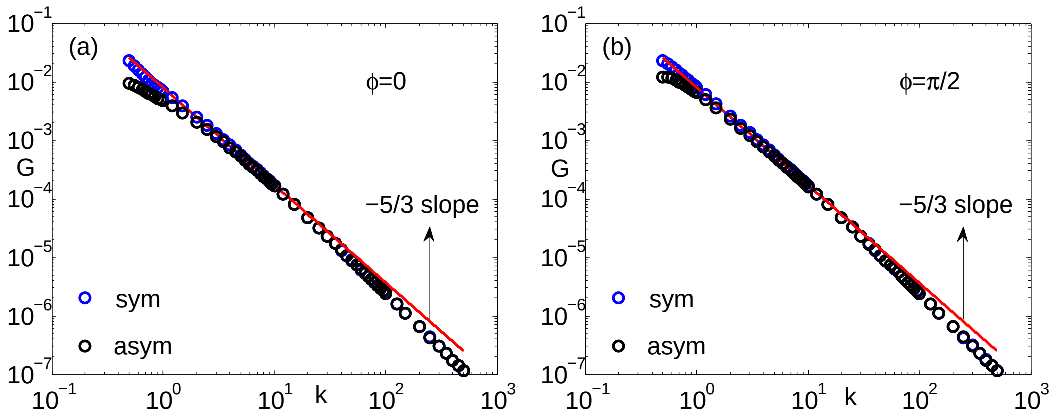

Figure 2. Spectrum of the amplification factor G of a collection of stable perturbation waves, $Re = 30$, $x_0 = 50$, $\alpha_i = 0$, symmetric (blue) and asymmetric (black) perturbations. (a) $\phi = 0$, (b) $\phi = \pi/2$. Red curves indicate $-5/3$ slope.

CONCLUSIONS

The experimental approach – based on the numerical determination of a large number of perturbations – here proposed to approximate the general perturbation solution of a Navier-Stokes field leads us to observe that, whether the waves are aligned with the base sheared flow or not, the energy of the intermediate range of wavenumbers in the spectrum decays with the same exponent ($-5/3$) that is observed in the spectrum of the velocity fluctuation of fully developed turbulent flows, where the nonlinear interaction is considered dominant. At the moment, we can conclude that the spectral power-law scaling of intermediate/inertial waves (with an exponent close to $-5/3$) is a general dynamical property of the Navier-Stokes solutions which encompasses the nonlinear interaction.

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